VANDERBILT
School *of* Engineering

DENVER, COLORADO
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Controllability Gramians Make Water Safer: Water Quality and Hydraulic Regulation in Drinking Networks

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Research Motivation

Drinking Water Networks

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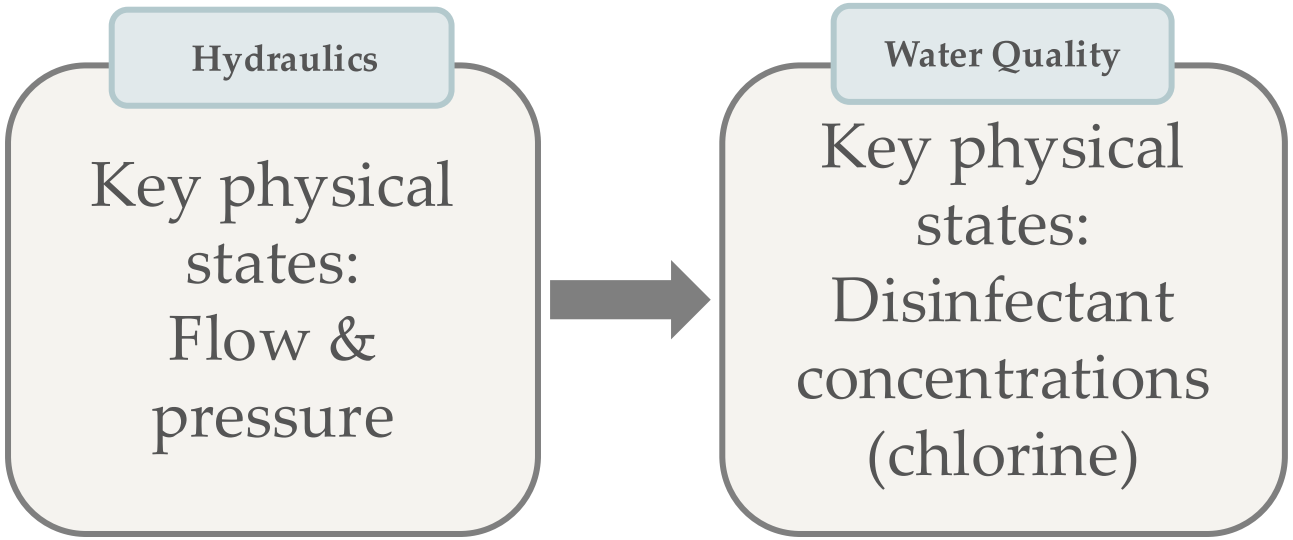
Objective:

Deliver water to consumers with desired water quantity-quality.

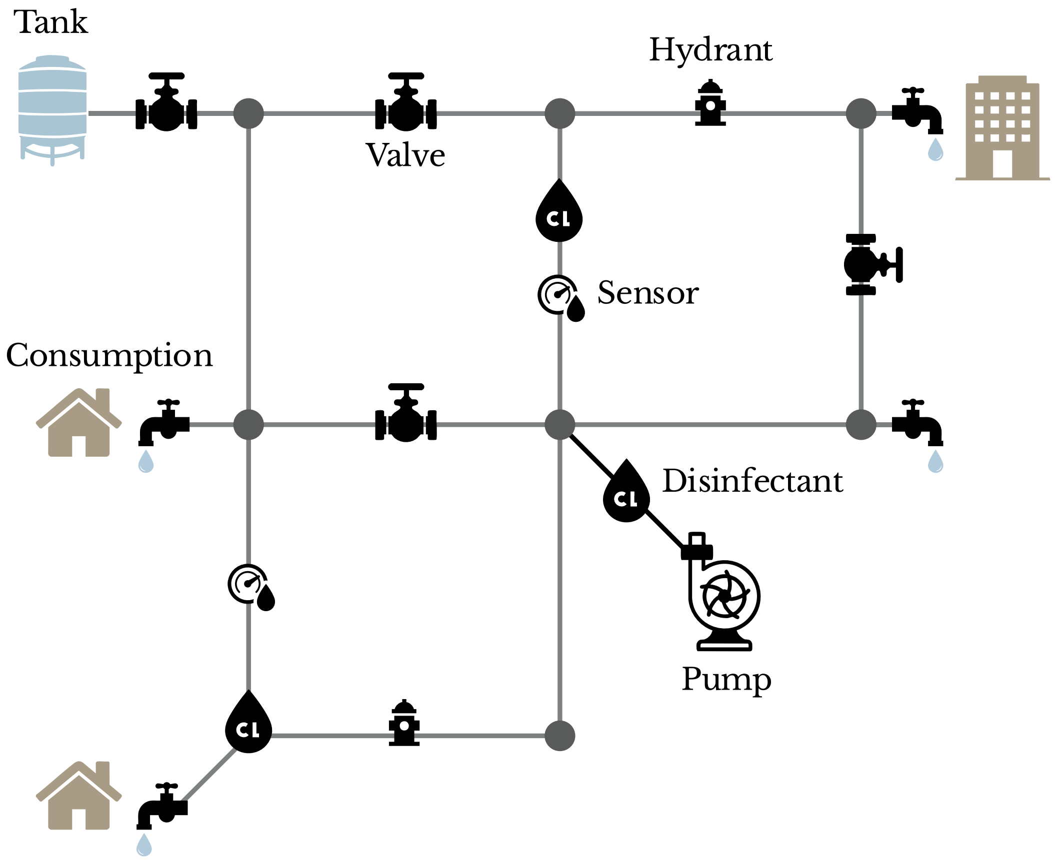


Hydraulics

Key physical states:
Flow & pressure

Water Quality

Key physical states:
Disinfectant concentrations (chlorine)



Joint Quantity-Quality Control Approach

Literature and Research Gap

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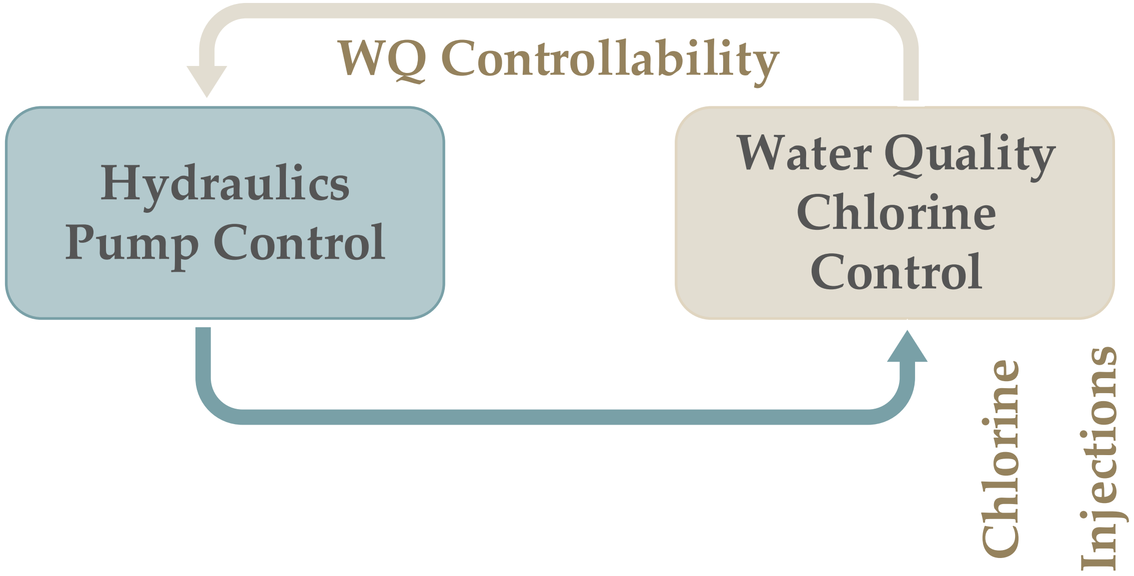
Studies have investigated integrating **quantity and quality** control problems.
(Drewa et al. 2007; Xie and Brddys 2015; Abdallah and Kapelan 2019)

Trade-offs between objectives:
no consideration of controllability of booster stations
or **reachability of the desired chlorine states**

Research Gap:

Designing a pump controls while attaining a certain level of **WQ controllability**, based on **integration of hydraulics and quality**



WQ Controllability

Hydraulics Pump Control

Water Quality Chlorine Control

Chlorine Injections

Hydraulics vs Water Quality Modeling

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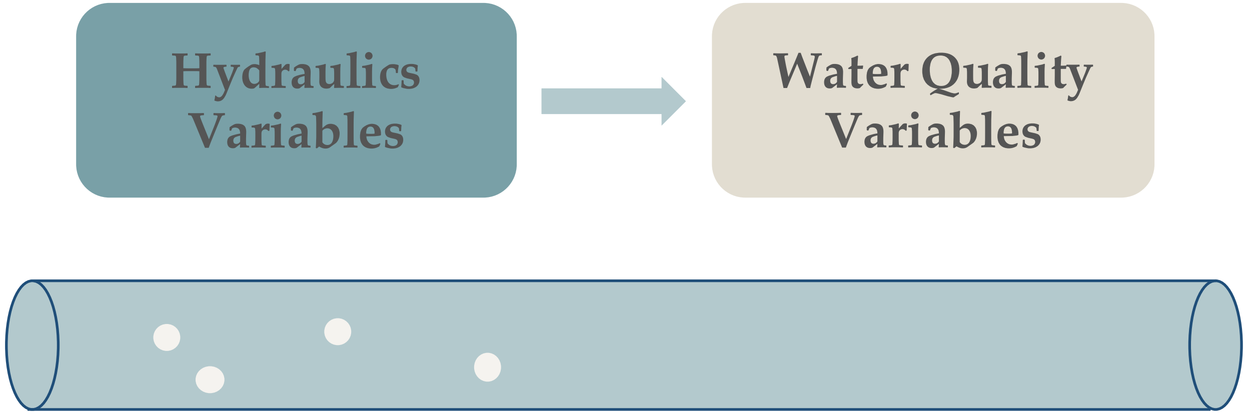
Time-scales

- Hydraulic time-step is **slower** within an **hourly** scale.
- Water Quality time-step is between **minutes** and **seconds**

Interdependency



Hydraulics Variables → **Water Quality Variables**

Control-Oriented Hydraulics Model

State-Space Representation

Reservoir Constant Head

$$h^{\mathrm{R}}(t+\Delta t_{\mathrm{H}}) = h^{\mathrm{R}}(t)$$

Junction Inflow – Outflow = Demand

$$\sum q_{\mathrm{in}}(t) - \sum q_{\mathrm{out}}(t) = q^{\mathrm{D}}(t)$$

Tank Change in Volume/Head ∝ Net Flow

$$h^{\mathrm{TK}}(t+\Delta t_{\mathrm{H}}) = h^{\mathrm{TK}}(t) + \alpha\Big(\sum q_{\mathrm{in}}(t) - \sum q_{\mathrm{out}}(t)\Big)$$

Pipe Head Loss due to Friction – Hazen-Williams Eq.

$$\Delta h^{\mathrm{P}}(t) = r q^{\mathrm{P}}(t)|q^{\mathrm{P}}(t)|^{0.852}$$

Pump Head Gain – **Control Component**

$$\Delta h^{\mathrm{M}}(t) = -s^2(h^0 - \alpha(s^{-1}q^{\mathrm{M}}(t))^{\nu})$$

Hyd-NDAE: $\boldsymbol{E}_w \boldsymbol{w}(t+\Delta t_{\mathrm{H}}) = \boldsymbol{A}_{\mathrm{H}} \boldsymbol{w}(t) + \Psi(\boldsymbol{w}, \boldsymbol{z}) + \boldsymbol{E}_D \boldsymbol{\Phi}_D$

Linear Terms

Nonlinear Terms

Junction Demands

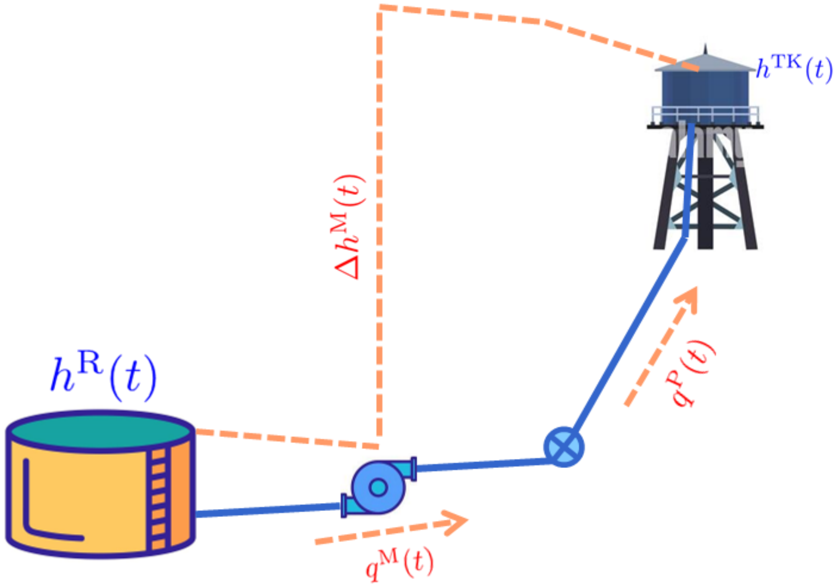
Heads

Flow Rates

States $\boldsymbol{w}(t) = \{\boldsymbol{h}(t), \boldsymbol{q}(t)\}$

Inputs $\boldsymbol{z}(t) = \{\boldsymbol{s}(t)\}$

Pump speed



Control-Oriented WQ Model

WQ State-Space Representation

- The chlorine reaction dynamics considered: **linear single-species decay** dynamics
- These PDEs are numerically **discretized** over **fixed spatio-temporal grid**

$$\frac{\partial c}{\partial t} = -v(t)\frac{\partial c}{\partial x} + R(c(x,t))$$

Nonlinear dynamics

$$\boldsymbol{E}(t)\boldsymbol{x}\left(t+\Delta t_{\mathrm{WQ}}\right) = \boldsymbol{A}_{\mathrm{WQ}}(t)\boldsymbol{x}(t) + \boldsymbol{B}_{\mathrm{WQ}}(t)\boldsymbol{u}(t)$$

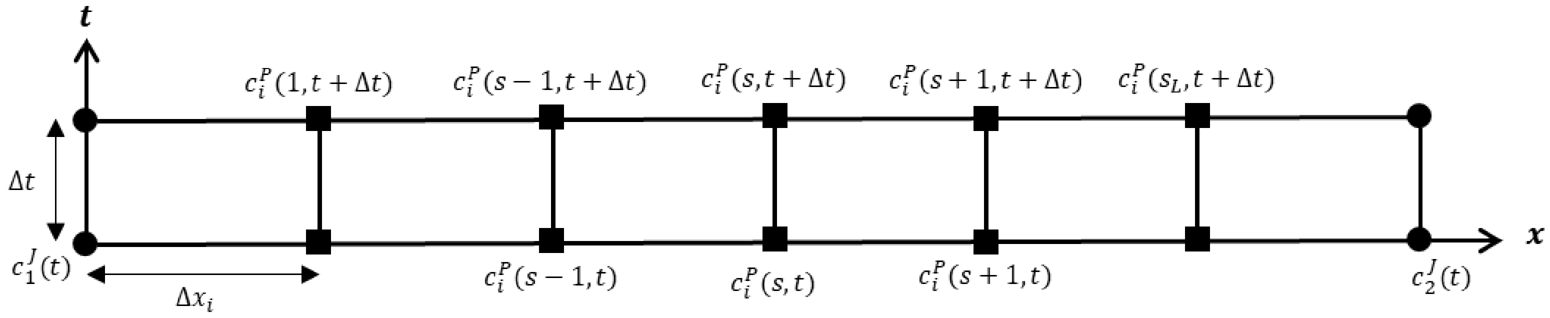
Chlorine at nodes

Chlorine in links

States $\boldsymbol{x}(t) = \{\boldsymbol{c}^{\mathcal{N}}(t), \boldsymbol{c}^{\mathcal{L}}(t)\}$

Inputs $\boldsymbol{u}(t) = \{\boldsymbol{c}^{\mathrm{B}}(t)\}$

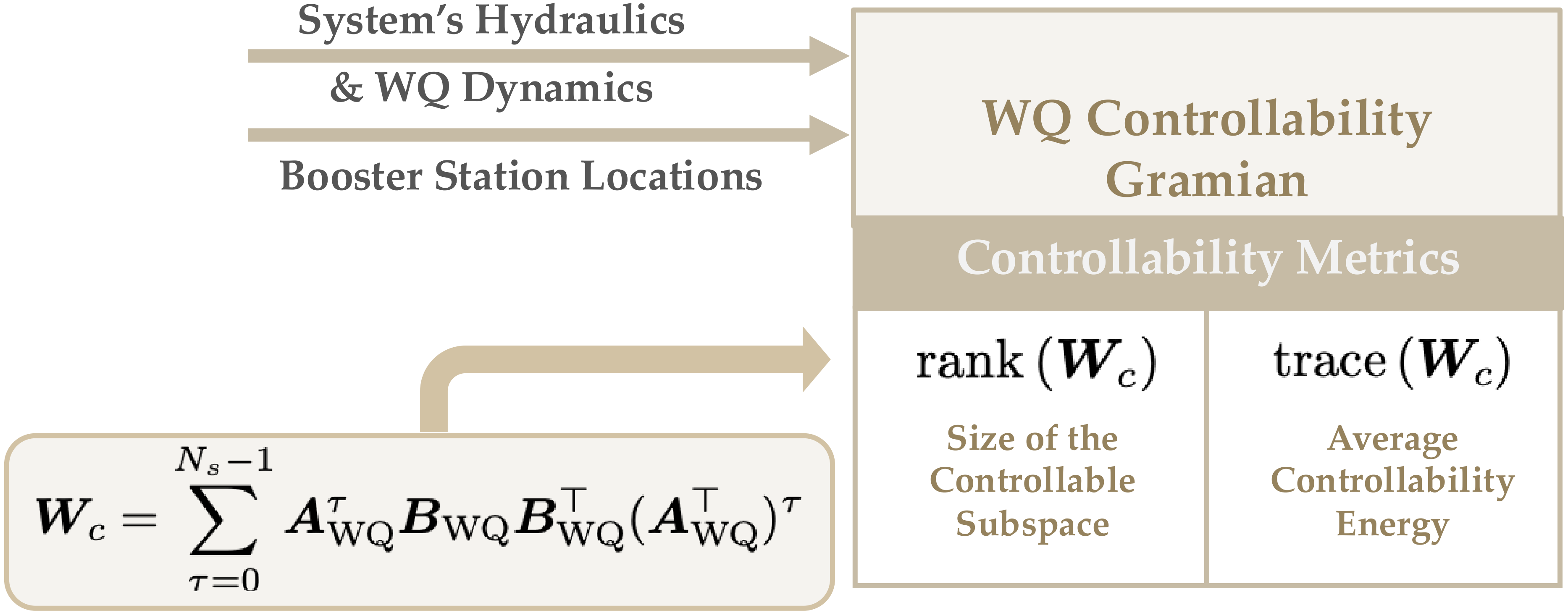
Booster Injections



Water Quality Controllability

WQ Control Gramian and Metrics

$$\boldsymbol{E}(t)\boldsymbol{x}\left(t+\Delta t_{\mathrm{WQ}}\right) = \boxed{\boldsymbol{A}_{\mathrm{WQ}}(t)}\boldsymbol{x}(t) + \boxed{\boldsymbol{B}_{\mathrm{WQ}}(t)}\boldsymbol{u}(t)$$



System's Hydraulics

& WQ Dynamics

Booster Station Locations

WQ Controllability Gramian

Controllability Metrics

$\operatorname{rank}\left(\boldsymbol{W}_c\right)$

Size of the Controllable Subspace

$\operatorname{trace}\left(\boldsymbol{W}_c\right)$

Average Controllability Energy

$$\boldsymbol{W}_c = \sum_{\tau=0}^{N_s-1} \boldsymbol{A}_{\mathrm{WQ}}^{\tau}\boldsymbol{B}_{\mathrm{WQ}}\boldsymbol{B}_{\mathrm{WQ}}^{\top}\left(\boldsymbol{A}_{\mathrm{WQ}}^{\top}\right)^{\tau}$$

Joint Quantity-Quality Control Approach

Decoupled Pump Control

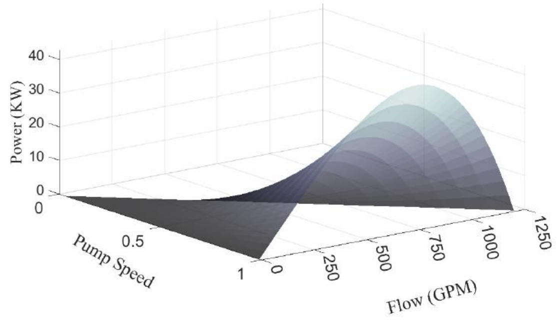
Hyd-NDAE: $\boldsymbol{E}_w \boldsymbol{w}\,(t + \Delta t_{\mathrm{H}}) = \boldsymbol{A}_{\mathrm{H}} \boldsymbol{w}(t) + \Psi(\boldsymbol{w}, \boldsymbol{z}) + \boldsymbol{E}_D \boldsymbol{\Phi}_D$

Minimize Power Consumption Cost by Pumps

$$\begin{aligned} \underset{\boldsymbol{w}(t), \boldsymbol{z}(t)}{\text{minimize}} \quad & \Pi(\boldsymbol{w}(t), \boldsymbol{z}(t)) \Delta t_{\mathrm{H}} \\ \text{subject to} \quad & \textbf{Hyd-NDAE}, \\ & \boldsymbol{w}_{\min} \leq \boldsymbol{w}(t) \leq \boldsymbol{w}_{\max}, \\ & \boldsymbol{0} \leq \boldsymbol{z}(t) \leq \boldsymbol{z}_{\max}. \end{aligned}$$

Nonlinear and Non-convex

Nonlinear and Non-convex



Joint Quantity-Quality Control Approach

Decoupled Pump Control

Hyd-NDAE-App:

$$\boldsymbol{E}_w \boldsymbol{w}\,(t+\Delta t_{\mathrm{H}}) = \widetilde{\boldsymbol{A}}_{\mathrm{H}}\boldsymbol{w}(t) + \boldsymbol{E}_\zeta \boldsymbol{\zeta}(t) + \widetilde{\Psi}(\boldsymbol{w}(t),\boldsymbol{z}(t)) + \boldsymbol{E}_D \boldsymbol{\Phi}_D$$

Minimize Power Consumption Cost by Pumps

$$\begin{aligned}
&\underset{\boldsymbol{w}(t),\boldsymbol{z}(t),\boldsymbol{\zeta}(t)}{\text{minimize}} && \Pi_{\mathrm{App}}(\boldsymbol{w}(t),\boldsymbol{z}(t),\boldsymbol{\zeta}(t))\Delta t_{\mathrm{H}}\\
&\text{subject to} && \text{Hyd-NDAE-App}\\
& && \boldsymbol{w}_{\min} \leq \boldsymbol{w}(t) \leq \boldsymbol{w}_{\max}\\
& && \boldsymbol{0} \leq \boldsymbol{z}(t) \leq \boldsymbol{z}_{\max}\\
& && \boldsymbol{\zeta}_{\min} \leq \boldsymbol{\zeta}(t) \leq \boldsymbol{\zeta}_{\max}
\end{aligned}$$

Quadratic and Convex

Quadratic and Convex

Joint Quantity-Quality Control Approach

Quality-Aware Pump Control

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Pump Control Problem + **WQ Controllability Metrics**

Rank-Informed

$$\underset{\boldsymbol{w}(t),\boldsymbol{z}(t),\boldsymbol{\zeta}(t)}{\text{minimize}} \ \Pi_{\text{App}}(\boldsymbol{w}(t),\boldsymbol{z}(t),\boldsymbol{\zeta}(t))\Delta t_{\text{H}} - \Theta \left\| \widetilde{\boldsymbol{W}}_c(\boldsymbol{w}(t),\boldsymbol{z}(t),\boldsymbol{\zeta}(t)) \right\|_*$$

$$\text{subject to} \ \ \text{Hyd-NDAE-App}$$

$$\boldsymbol{w}_{\text{min}} \leq \boldsymbol{w}(t) \leq \boldsymbol{w}_{\text{max}}$$

$$\boldsymbol{0} \leq \boldsymbol{z}(t) \leq \boldsymbol{z}_{\text{max}}$$

$$\boldsymbol{\zeta}_{\text{min}} \leq \boldsymbol{\zeta}(t) \leq \boldsymbol{\zeta}_{\text{max}}$$

$$l_r \left\| \widetilde{\boldsymbol{W}}_c \right\|_2 - \text{trace}\left(\widetilde{\boldsymbol{W}}_c\right) \leq 0.$$

Joint Quantity-Quality Control Approach

Quality-Aware Pump Control

Pump Control Problem + **WQ Controllability Metrics**

Energy-Driven

$$\underset{\boldsymbol{w}(t),\boldsymbol{z}(t),\boldsymbol{\zeta}(t)}{\text{minimize}} \; \Pi_{\text{App}}(\boldsymbol{w}(t),\boldsymbol{z}(t),\boldsymbol{\zeta}(t))\Delta t_{\text{H}} \boxed{- \Theta \, \text{trace}\left(\widetilde{\boldsymbol{W}}_c(\boldsymbol{w}(t),\boldsymbol{z}(t),\boldsymbol{\zeta}(t))\right)}$$

$$\text{subject to} \;\; \text{Hyd-NDAE-App}$$

$$\boldsymbol{w}_{\min} \leq \boldsymbol{w}(t) \leq \boldsymbol{w}_{\max}$$

$$\boldsymbol{0} \leq \boldsymbol{z}(t) \leq \boldsymbol{z}_{\max}$$

$$\boldsymbol{\zeta}_{\min} \leq \boldsymbol{\zeta}(t) \leq \boldsymbol{\zeta}_{\max}$$

$$\boxed{\text{trace}\left(\widetilde{\boldsymbol{W}}_c\right) \geq 0.}$$

Case Study: Joint Quantity—Quality Control

Decoupled vs. WQ Controllability-Aware Optimal Pump Scheduling

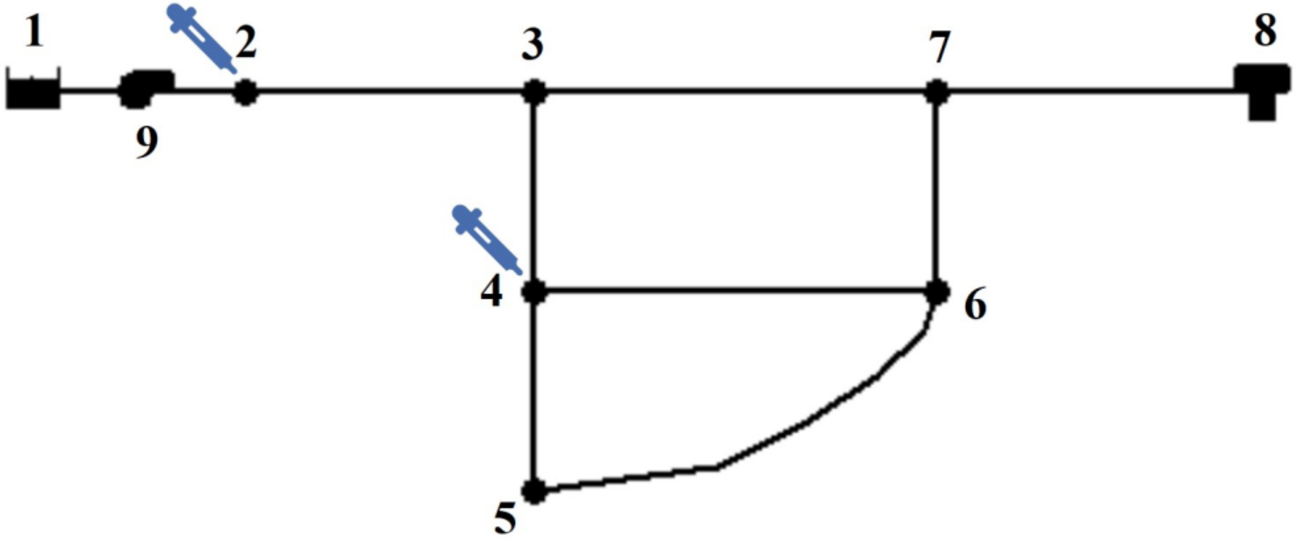
System Setup

- Chlorination at **Junctions 2 and 4**
- 1 Reservoir, 1 Tank, 1 Pump, 6 Junctions

Parameters

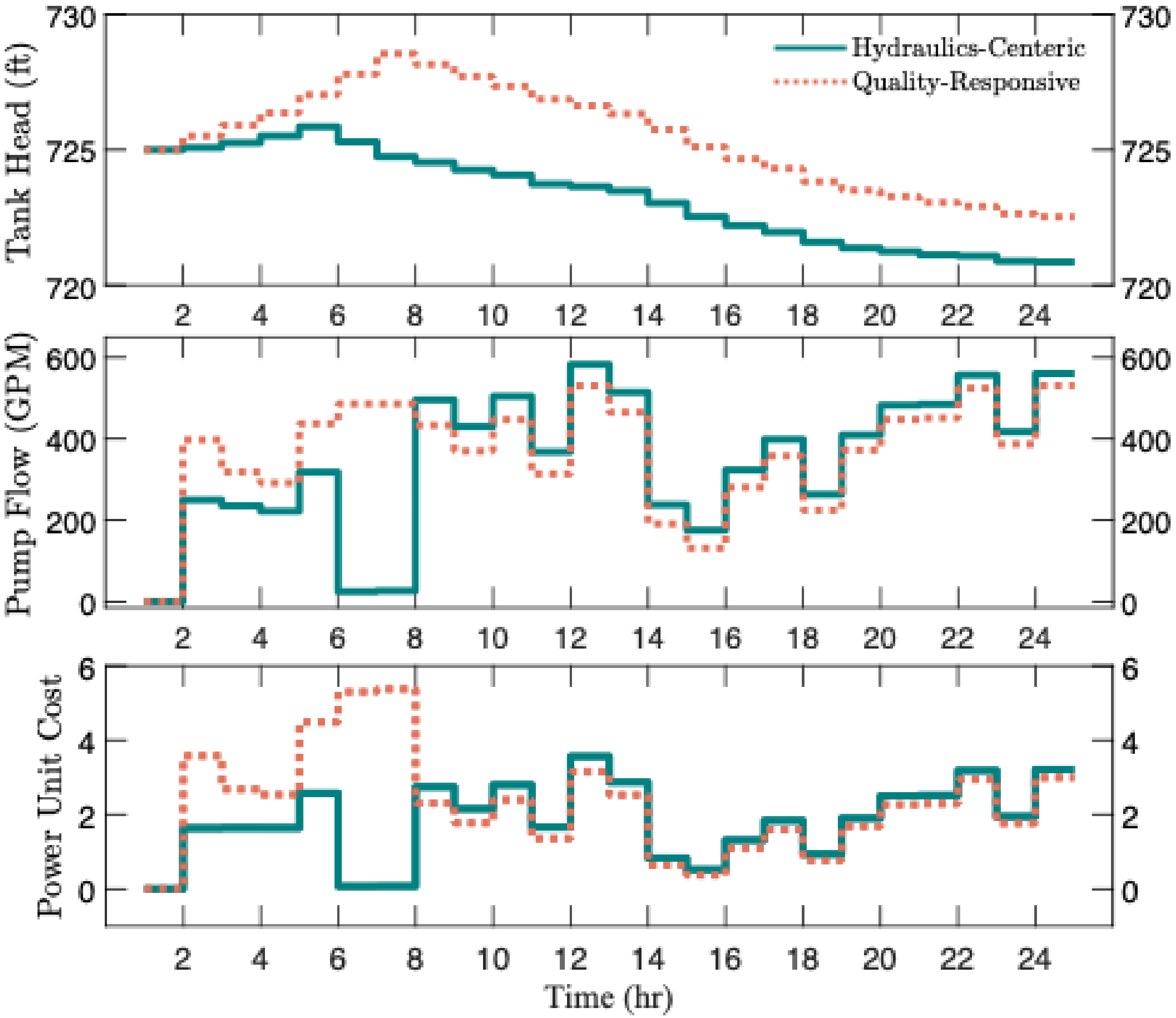
- Time-steps
  1 min WQ time-step
  30 min Hydraulic time-step
- Initial chlorine concentrations:
  - All components set to **0.2 mg/L**
  - Reservoir 1 set to **0.8 mg\L**

Layout



Case Study: Joint Quantity—Quality Control

Decoupled vs. WQ Controllability-Aware Optimal Pump Scheduling



Quantity-Responsive

- **Insufficient chlorine** residuals at nodes far from tank
- **Pump activated** when tank reaches min levels

Quality-Responsive

- **Same overall** pump schedule pattern
- **Increased initial** Pumping to favor **Chlorine mixing and reachability**
- **15% increase** in pump energy cost, while **achieving WQ levels**.
- Chlorine injections are **balanced** among **booster stations**

Concluding Remarks

Decoupled vs. WQ Controllability-Aware Optimal Pump Scheduling

- Improved chlorine control performance across **different network configurations and demand patterns**
- Ensures that the WDN has **enough chlorine residuals**
- Chlorine injections are **balanced** among **booster stations**
- Variations in **consumer demand patterns,** leads to substantial **shifts in system hydraulics** to maintain WQ levels

Future Directions

Addressing Limitations and Possible Expansions

- Established coupled water quantity-quality model ready for plug-in control algorithms such as MPC, actuator placement, etc.
- Extend to distributed robustness-guaranteed control with energy- and disinfectant-aware optimization and sensor/actuator placement via controllability metrics.

Questions?

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