VANDERBILT
School *of* Engineering

DENVER, COLORADO
July 8-10th, 2025

Generalizable Stability Metrics For Power Grids

Mohamad H. Kazma and Ahmad F. Taha

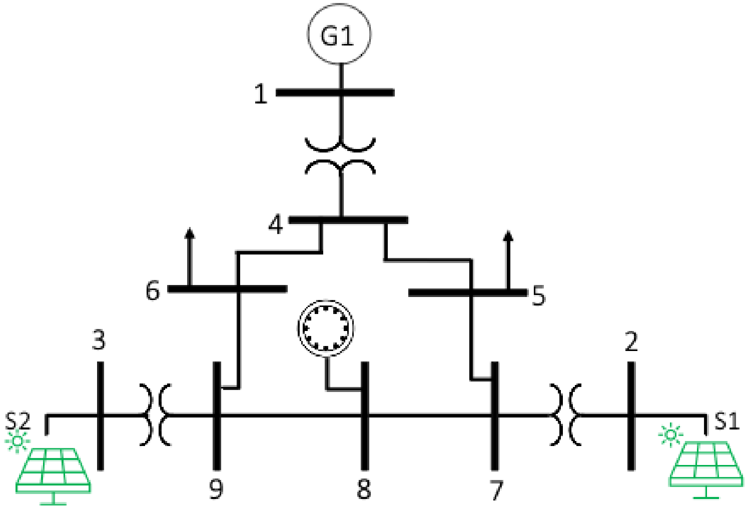
WeB07.1 – Power Grids – July 9th, 2025

Power Grids under Renewable Integration

Stability and Uncertainty from Loads

Power systems inundated with intermittent and uncertain loads that affect **system stability**

Ability to regain a state of operating equilibrium after a physical disturbance or perturbation

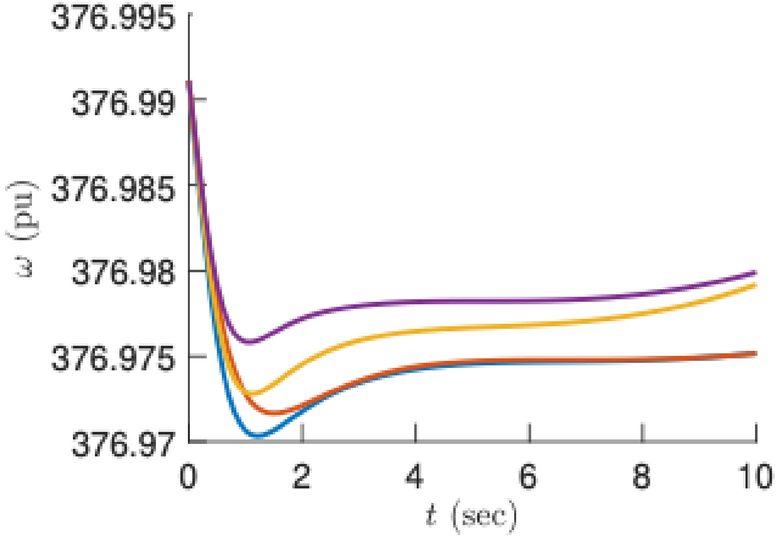


Direct stability method (Lyapunov Energy Function)

Quantifying an energy function that is indicative of system stability *(Y.J. Isbieh et al., 2019 (IET Smart Grid))*

Lyapunov Exponents (LEs) Method

Characterizing infinitesimal separation rates of system trajectories *(Bosetti and Khan, 2018 (IEEE Trans. Power Systems))*



Quantifying Stability of Power Grids

Literature on Stability: LEs from power system's perspective

DENVER, COLORADO
July 8-10th, 2025

First adapted from fields of chaos and ergodic theory

(Liu et al, 1994 (IEEE Trans. Power Systems))

Considers one type of stability (no uncertainty propagation)

Swings that lead to rotor angle instability – Yan et al., 2011 (IEEE Trans. Power Systems)

PMU-based approach for stability (measurement noise and state-estimation errors)

Relies on state-estimation

- voltage stability – Dasgupta et al., 2013 (IEEE Trans. Power Systems)
- rotor angle stability – Wei et al., 2018 (IEEE Trans. Power Systems)

Either model-free or simplified dynamics (no differential and algebraic state coupling)

Relies on linear or decoupled ODE models of power networks

- Rotating machines modeled by the swing equation – Bosetti and Khan, 2018 (IEEE Trans. Power Systems)

Contributions Towards Power System Stability

An LE-based Approach for Stability Quantification and Load Allocation

DENVER, COLORADO
July 8-10th, 2025

Quantifying overall system stability and identifying stable nodes for renewables resources allocation

- **Generalized system stability**
  Considers frequency, voltage and rotor angle stability
- **Uncertainty propagation**
  Quantifies the uncertainty the propagates from any load perturbation
- **Informs on the practical allocation of renewables and much more**
  A submodular maximization approach that enables stable node identification

Road Towards Generalized Stability Metrics

Step 1 – Defining the NDAE Power System Model

1 NDAE Form

General Nonlinear Differential Algebraic Equations (NDAEs)

Diff. States — Alg. States

Gen. Dynamics: $$\dot{\boldsymbol{x}}_d = \boldsymbol{f}(\boldsymbol{x}_d, \boldsymbol{x}_a, \boldsymbol{u})$$ — Inputs

Alg. Constraints: $$\boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x}_d, \boldsymbol{x}_a)$$

generator dynamics:

$$\dot{\delta}_i = \omega_i - \omega_0,$$

$$M_i \dot{\omega}_i = T_{\mathrm{M}i} - P_{\mathrm{G}i} - D_i(\omega_i - \omega_0),$$

$$T'_{\mathrm{d0}i} \dot{E}'_i = -\frac{x_{\mathrm{d}i}}{x'_{\mathrm{d}i}} E'_i + \frac{x_{\mathrm{d}i} - x'_{\mathrm{d}i}}{x'_{\mathrm{d}i}} v_i \cos(\delta_i - \theta_i) + E_{\mathrm{fd}i},$$

$$T_{\mathrm{CH}i} \dot{T}_{\mathrm{M}i} = T_{\mathrm{M}i} - \frac{1}{R_{\mathrm{D}i}}(\omega_i - \omega_0) + T_{\mathrm{r}i},$$

algebraic constraints:

$$P_{\mathrm{G}i} = \frac{1}{x'_{\mathrm{d}i}} E'_i v_i \sin(\delta_i - \theta_i) - \frac{x_{\mathrm{q}i} - x'_{\mathrm{d}i}}{2x'_{\mathrm{d}i} x_{\mathrm{q}i}} v_i^2 \sin(2(\delta_i - \theta_i)),$$

$$Q_{\mathrm{G}i} = \frac{1}{x'_{\mathrm{d}i}} E'_i v_i \cos(\delta_i - \theta_i) - \frac{x_{\mathrm{q}i} - x'_{\mathrm{d}i}}{2x'_{\mathrm{d}i} x_{\mathrm{q}i}} v_i^2 - \frac{x_{\mathrm{q}i} - x'_{\mathrm{d}i}}{2x'_{\mathrm{d}i} x_{\mathrm{q}i}} v_i^2 \cos(2(\delta_i - \theta_i)).$$

$$P_{\mathrm{G}i} + P_{\mathrm{L}i} + P_{\mathrm{R}i} = \sum_{j=1}^{N} v_i v_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}),$$

$$Q_{\mathrm{G}i} + Q_{\mathrm{L}i} + Q_{\mathrm{R}i} = \sum_{j=1}^{N} v_i v_j (G_{ij} \cos\theta_{ij} - B_{ij} \sin\theta_{ij}),$$

Road Towards Generalized Stability Metrics

Step 2 – Discretization and ODE Transformation of Power Model

DENVER, COLORADO
July 8-10th, 2025

1 NDAE Form

2 Discrete ODE Form

Kazma and Taha, 2024

Transformed Discrete Differential Algebraic Equations

Rewrite in discrete-time nonlinear ODE formulation

Differential and algebraic dynamics

$$\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \tilde{\boldsymbol{f}}\left(\boldsymbol{x}_{k-1}\right)$$

$$\boldsymbol{x}_k = \left[\boldsymbol{x}_{d,k},\ \boldsymbol{x}_{a,k}\right]^\top$$

Road Towards Generalized Stability Metrics

Step 3 – From Nonlinear ODE to Variational Form

DENVER, COLORADO
July 8-10th, 2025

1 NDAE Form

2 Discrete ODE Form

3 Variational Form

Discrete Variational System Dynamics

$$\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \tilde{\boldsymbol{f}}\left(\boldsymbol{x}_{k-1}\right)$$

Variational Form

$$\boldsymbol{\delta x}_k = \boldsymbol{\Phi}_0^k\left(\boldsymbol{x}_0\right)\boldsymbol{\delta x}_0$$

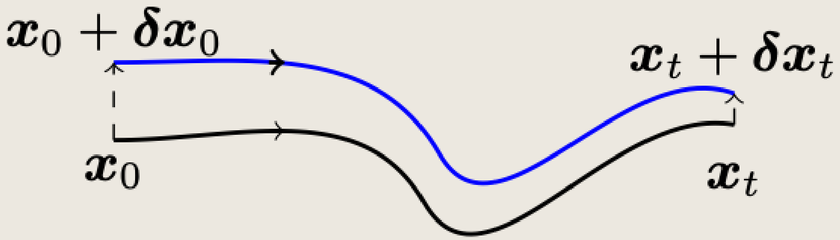
Variational Mapping Function

$$\boldsymbol{\Phi}_0^k = \boldsymbol{\Phi}_{k-1}^k \boldsymbol{\Phi}_{k-2}^{k-1} \dots \boldsymbol{\Phi}_0^1 \boldsymbol{\Phi}_0^0 = \prod_1^{i=k} \boldsymbol{\Phi}_{i-1}^i$$

Computed along using chain rule

Simply
follows the system trajectory
along the tangent space

Kazma and Taha, 2024



Road Towards Generalized Stability Metrics

Step 4 – Lyapunov Exponents and Stability

1 NDAE Form

2 Discrete ODE Form

3 Variational Form

4 LEs Computation

Computing LEs for exponential stability

Maximum LE

$$\lambda_{\max} = \max\{\frac{1}{N} \log \frac{\|\boldsymbol{\delta x}_k\|}{\|\boldsymbol{\delta x}_0\|}\}$$

Spectrum of LEs

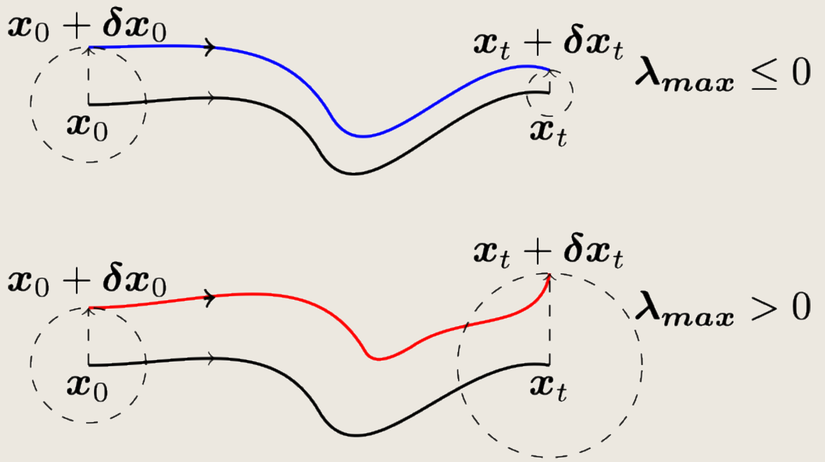
$$\boldsymbol{\lambda} = \frac{1}{N} \log\|\boldsymbol{\Phi}_0^k\|$$

- *Stable or periodic* if

$$\lambda_{\max} \leq 0$$

- *Unstable* if

$$\lambda_{\max} > 0$$



Road Towards Generalized Stability Metrics

Step 5 – State-parameterization for Stability Quantification

DENVER, COLORADO
July 8-10th, 2025

1. NDAE Form
2. Discrete ODE Form
3. Variational Form
4. LEs Computation
5. Stability Parameterization

Generalized stability quantification from load perturbations

Parameterization Parameter

Generator Buses

$$\lambda_i = \begin{cases} \lambda_{i_{\mathcal{G}}} = \frac{1}{2N} \log\left(\boldsymbol{\Phi}_0^k(\boldsymbol{\gamma})^\top \boldsymbol{\Phi}_0^k(\boldsymbol{\gamma})\right), & \gamma_j \in \{\delta_i, \omega_i, v_i\}_{i\,\in\,\mathcal{G}}, \\ \lambda_{i_{\mathcal{L}}} = \frac{1}{2N} \log\left(\boldsymbol{\Phi}_0^k(\boldsymbol{\gamma})^\top \boldsymbol{\Phi}_0^k(\boldsymbol{\gamma})\right), & \gamma_j \in \{v_i\}_{i\,\in\,\mathcal{L}}, \end{cases}$$

Load Buses

Stability measures	
Generators	Load Buses
Rotor angle Frequency Voltage	Voltage

Extended measures
Renewables Integrated
Inverter angles of PV plants Voltages across the AC capacitor

Road Towards Generalized Stability Metrics

Step 6 – Proposed Stability and RER Formulation

1 NDAE Form

2 Discrete ODE Form

3 Variational Form

4 LEs Computation

5 Stability Parameterization

6 Stable Node Identification and RER Allocation

LE-based stability measure

Theorem 1

LE-based

$$\log\det(\mathbf{\Phi}_0^k(\gamma)^\top \mathbf{\Phi}_0^k(\gamma)) \equiv \beta \sum_{i=1}^{N} \lambda_i, \quad \forall\, i \in \mathcal{N}$$

Uncertainty Propagation

Overall impact from a load perturbation at any bus

Does not require Eigenvalue computations

Road Towards Generalized Stability Metrics

Step 6 – Proposed Stability and RER Formulation

DENVER, COLORADO
July 8-10th, 2025

1 NDAE Form

2 Discrete ODE Form

3 Variational Form

4 LEs Computation

5 Stability Parameterization

6 Stable Node Identification and RER Allocation

Renewables allocation within power networks

$$\underset{\mathcal{S}}{\text{maximize}} \;\; \mathcal{L}(\mathcal{S}) = \log\det(\boldsymbol{\Phi}_0^k(\boldsymbol{\gamma})^\top \boldsymbol{\Phi}_0^k(\boldsymbol{\gamma}))$$

$$\text{subject to } |\mathcal{S}| = s, \; \mathcal{S} \subseteq \mathcal{V}$$

Set of renewables to allocate

Submodularity Property

Renders computational scalable Allocation

Case Studies – 4^{th} order Power Grid

System Setup and Parameters

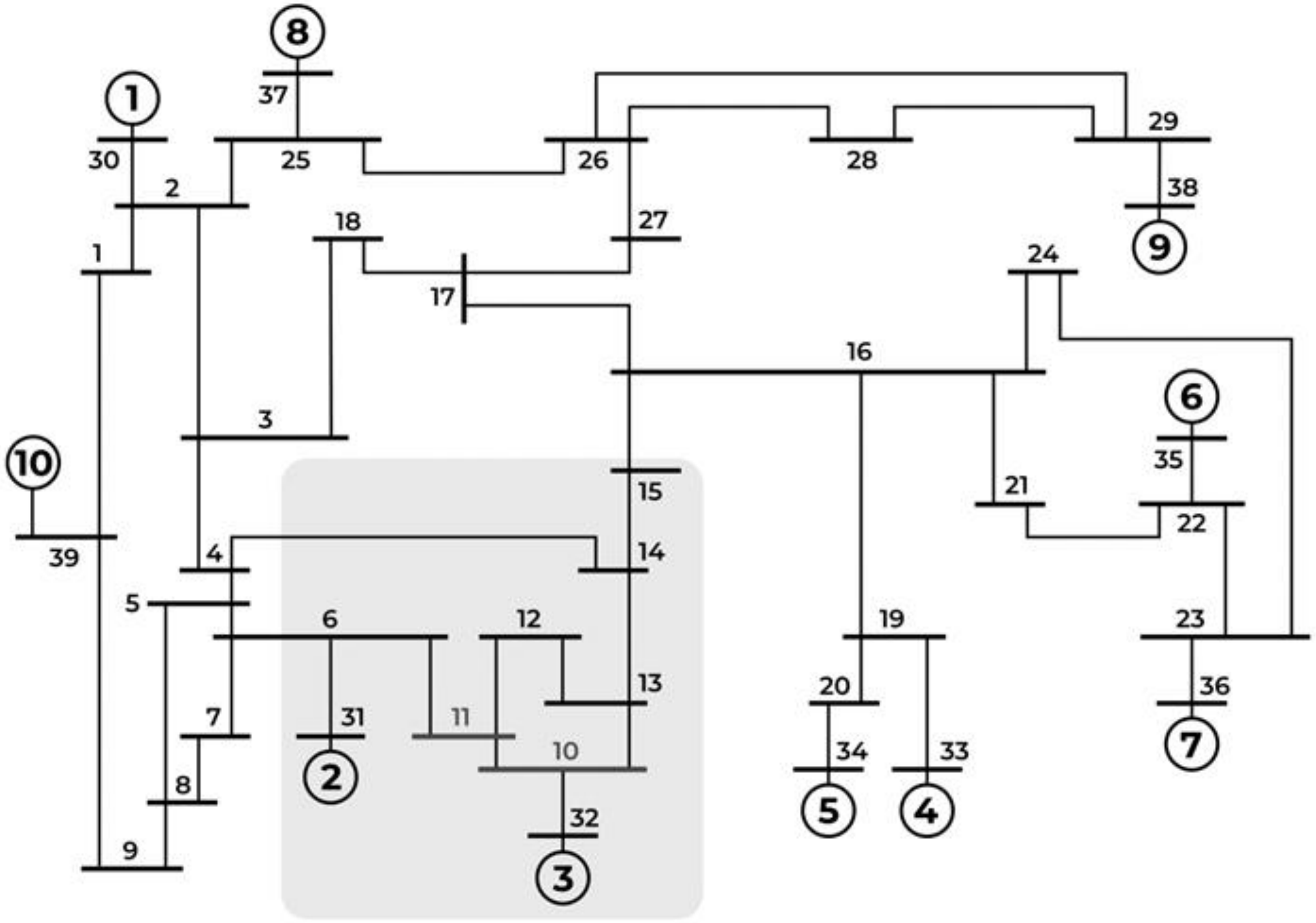
System Setup

- 39 Buses, 10 generators
- Synchronous speed
  - $\omega_0 = 120\pi \text{ rad/sec}$

Parameters

- **10** seconds time-step
- Load Perturbations (real and reactive power:
  - $(\mathrm{P}_i^0, \mathrm{Q}_i^0)$ at bus $i \in \mathcal{N}$
    - $(\tilde{\mathrm{P}}_\mathrm{i}^0, \tilde{\mathrm{Q}}_\mathrm{i}^0) = \left(1 + \frac{\beta}{100}\right)(\mathrm{P}_\mathrm{i}^0, \mathrm{Q}_\mathrm{i}^0)$

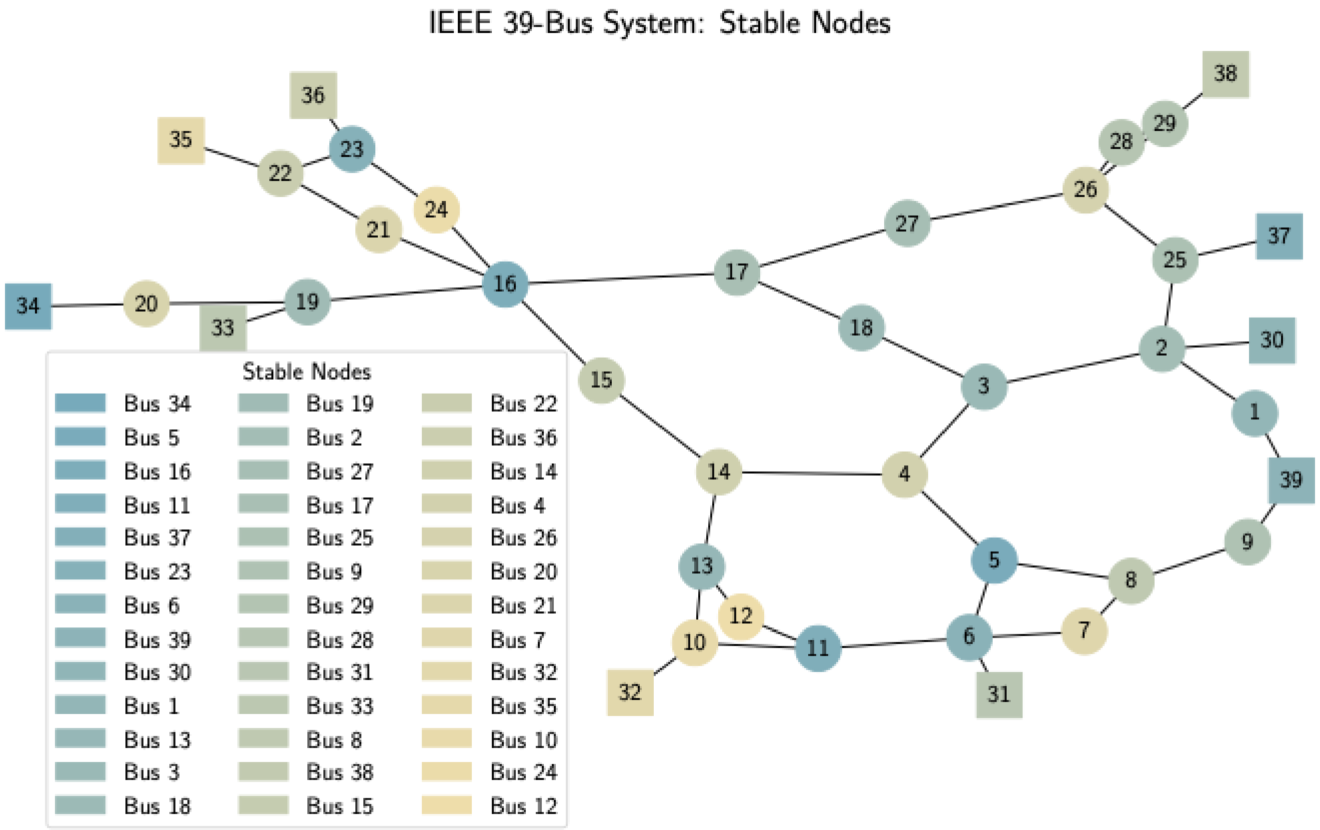
IEEE Case 39



Stable Node Identification

Node Stability and RER Allocation

Ranked buses according to the computed stability metrics (frequency, voltage, rotor angle).



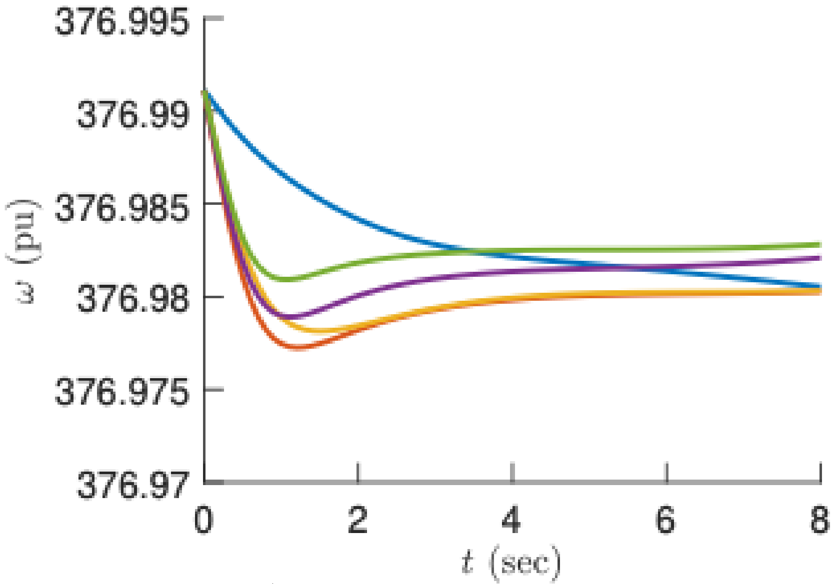
Outcomes

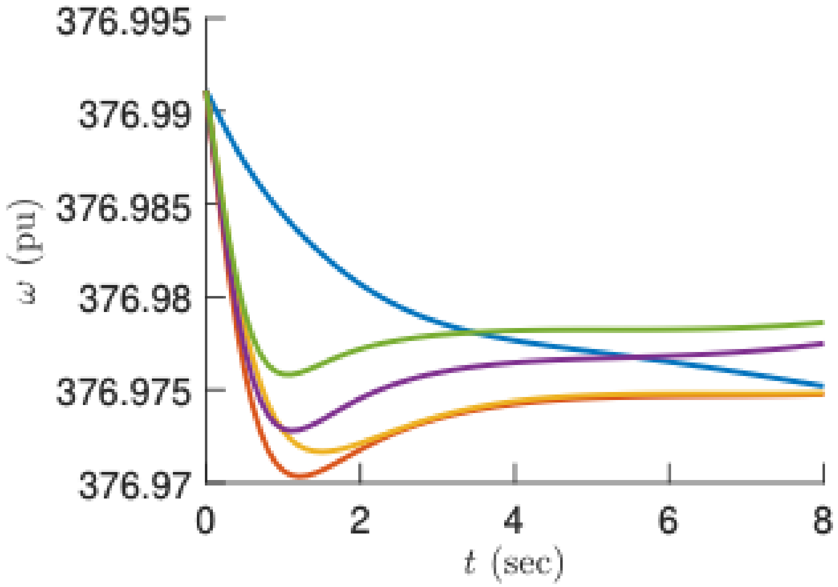
- *Ranked node/buses* based on stability metrics
- *Computationally inexpensive* due to *submodular* problem formulation
- *Robust* to change in operating conditions

Stable Node Identification

Uncertainty Propagation within the Power Grid

Clearing time for frequency transients induced by allocating an uncertain load at Bus 12 at the most stable Bus 34.





Uncertainty propagation is less severe from load at **most stable bus**

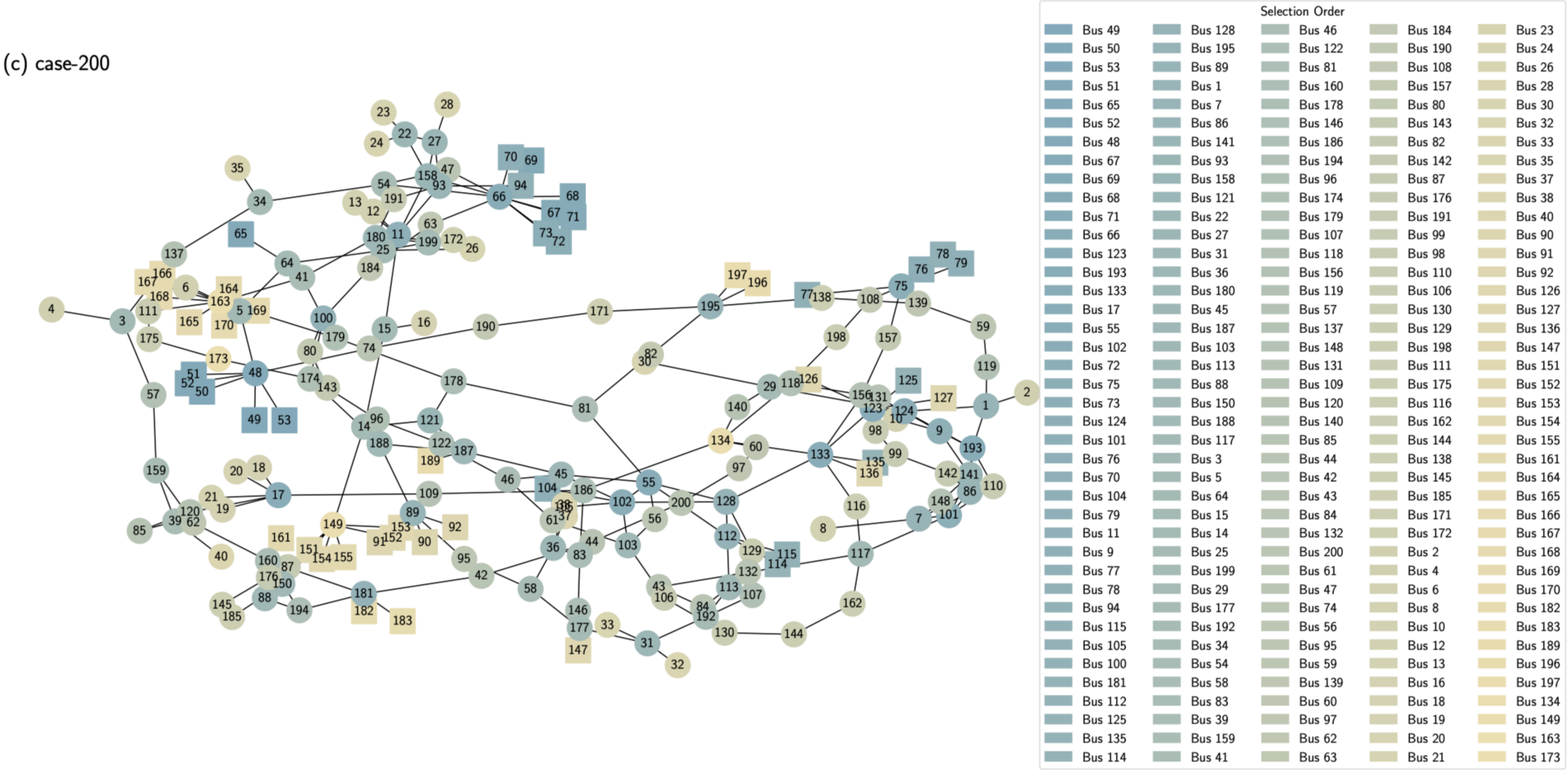
Same initial load perturbation

Amplitude and clearing time of a load perturbation that propagates to the generators

Stable Node Identification

Extended to Larger Systems

DENVER, COLORADO
July 8-10th, 2025



Concluding Notes

On the stability from a LEs perspective

Generalized stable node and RER allocation method framework

Overall Stability Metric
A LE-based stability metric that ranks nodes according to type of stability considered

Extends to RER integrated models of power systems
The stability measures can include other stability modes depending on type of bus

RER Allocation
A submodular allocation framework for identifying buses where RERs have least impact on overall stability

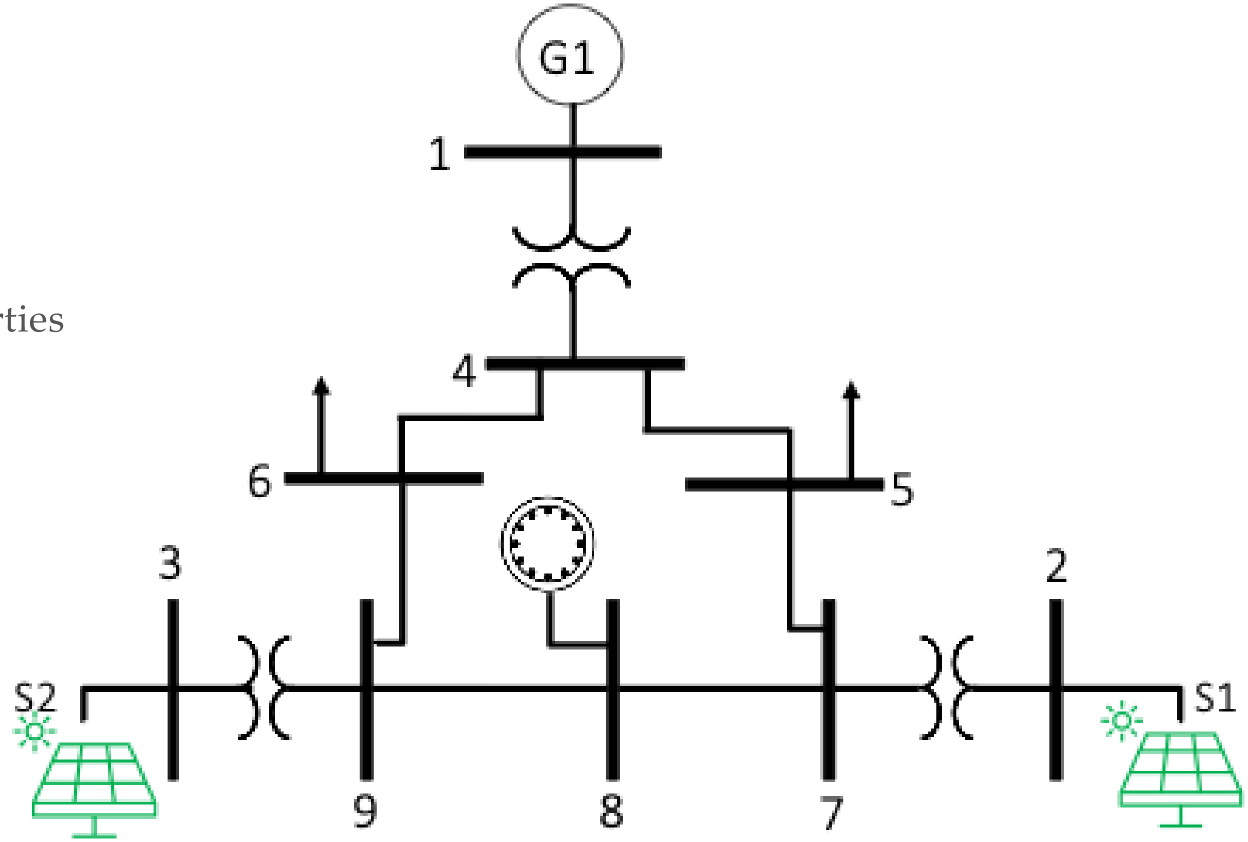
Informs Operators
A method to inform system operators on the extent to which a load perturbation propagates to other buses

LE Equivalence
Provided evidence that log-det of state-deformation of variational form are equivalent to LEs of the system

Future Work and Broader Impacts

On the stability from a LEs perspective

- **Incorporating dynamic models of RERs**
  Such as PV plants and solar farms
- **Real-world scale power grids**
  Verification on large scale power networks
- **Can be used for PMU or inertia allocation**
  Can be used to characterize other types of network properties



DENVER, COLORADO
July 8-10th, 2025

Reference List

DENVER, COLORADO
July 8-10th, 2025

- H. Bosetti and S. Khan, "Transient Stability in Oscillating Multi-Machine Systems Using Lyapunov Vectors," IEEE Transactions on Power Systems, vol. 33, no. 2, pp. 2078-2086, 2018.

- Y. J. Isbeih, M. S. E. Moursi, W. Xiao, and E. El-Saadany, "Generator-based threshold for transient stability assessment," IET Smart Grid, vol. 2, no. 3, pp. 407-419, 2019.

- C. W. Liu, J. S. Thorp, J. Lu, R. J. Thomas, and H. D. Chiang, "Detection of Transiently Chaotic Swings in Power Systems using Real-Time Phasor Measurements," IEEE Transactions on Power Systems, vol. 9, no. 3, pp. 1285-1292, 1994.

- J. Yan, C. C. Liu, and U. Vaidya, "PMU-based monitoring of rotor angle dynamics," IEEE Transactions on Power Systems, vol. 26, no. 4, pp. 2125-2133, 2011.

- S. Dasgupta, M. Paramasivam, U. Vaidya, and V. Ajjarapu, "Real-time monitoring of short-term voltage stability using PMU data," IEEE Transactions on Power Systems, vol. 28, no. 4, pp. 3702-3711, 2013.

- S. Wei, M. Yang, J. Qi, J. Wang, S. Ma, and X. Han, "Model-Free MLE Estimation for Online Rotor Angle Stability Assessment with PMU Data," IEEE Transactions on Power Systems, vol. 33, no. 3, pp. 2463-2476, 2018.

Questions?

Generalizable Stability Metrics for Power Grids

Mohamad H. Kazma
mohamad.h.kazma@vanderbilt.edu
July 9th, 2025